$$\delta W/\delta S = (1-\nu)\,\pi\mu^{-1}(K_2^0)^2\,\{1+\varepsilon^2[-3\lambda/8+17/16+(1-4\nu)^2/27]+o(\varepsilon^2)\}$$

In conclusion, we note that the action of even a small additional external pressure $p=\varepsilon K_p p_z$ on a crack edge results in a change in the superposition domain. By reasoning analogous to that presented above we obtain that the superposition domain is defined by the inequalities $-1 \leqslant M \leqslant K$, where $K=-{}^1/_3 - 8K_p/[3(4\nu-1)]$ and for the exposure we have

$$\varkappa_r \approx \begin{cases} {}^1/_2(1-\nu)\mu^{-1}(4\nu-1)\,p_z h\sqrt{1-M}\,(M-K)^{3/2}, & K \leqslant M \leqslant 1 \\ 0, & -1 \leqslant M \leqslant K \end{cases}$$

For $p > \varepsilon K_p^1 p_z$, $K_p^1 = \nu - {}^1/_4$ the crack is completely open, and completely closed for $p < \varepsilon K_p^2 p_z$, $K_p^2 = -2(\nu - {}^1/_4)$.

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GREEN'S FUNCTION FOR THE BENDING OF A PLATE ON AN ELASTIC HALF-SPACE*

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Improper Woinowski-Krieger integrals /1/ expressing the deflections of an infinite plate and the contact reactions of an elastic half-space subjected to a unit normal force are considered. Elementary formulas to calculate the quantities mentioned in the neighbourhood of the point of application of the load are obtained from the power series expansion with a logarithm by Watson's method. The results of calculations using these formulas are in good agreement with the results of a numerical integration of quadratures /2/. The analytical representations obtained for Green's functions are convenient for utilization as kernels of the integral equations when solving contact problems for the interaction of bodies, one of which is reinforced by a thin covering.

Under the action of a unit normal force at a point with coordinates (x_1, y_1) on an infinite plate lying without friction and adhesion on an elastic half-space, the deflections w and contact pressures p at a point with coordinates (x, y) are expressed by the integrals /1, 3/

$$w = l^2(2D)^{-1}w_0,\quad p = l^{-2}p_0 \tag{1}$$

$$w_0 = \frac{1}{\pi}\int_0^\infty \frac{J_0(\lambda\rho)}{\lambda^3+1}\,d\lambda,\qquad p_0 = \frac{1}{2\pi}\int_0^\infty \frac{J_0(\lambda\rho)\,\lambda}{\lambda^3+1}\,d\lambda$$

$$\rho = l^{-1}((x-x_1)^2+(y-y_1)^2)^{1/2},\quad D = Eh^3(12(1-\nu^2))^{-1}$$

$$l = (2DE_0^{-1}(1-\nu_0^2))^{1/3}$$

Here E, ν are the elastic modulus and Poisson's ratio of a plate of thickness h while

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E_0, ν_0 are the characteristics of the elastic half-space, and $J_0(z)$ is the Bessel function.

To speed up the convergence, the slowly convergent part was extracted from the expressions for w_0 and p_0 and represented in terms of tabulated Thomson functions /4/. The quadratures were here conserved in the solution, but their convergence was improved. An analytic calculation of the improper integrals, based on Watson's method which turned out to be effective in evaluating Fourier integrals representing the fundamental solutions of the differential equations for shallow shells /5, 6/, is proposed in this paper.

Let us consider the evaluation of w_0 in detail. We will use the Mellin-Burns integral representation for the Bessel functions (/7/, p.30). We set $\nu = 0, s = -z, c = -\sigma$ therein and transform the numerator of the integrand by using the properties of the gamma function /8/. We obtain

$$J_0(\lambda\rho) = \frac{1}{4i}\int_{\sigma+i\infty}^{\sigma-i\infty} \frac{(^1/_2\lambda\rho)^z\,dz}{\Gamma^2(1+{}^1/_2z)\sin{}^1/_2\pi z} \tag{2}$$

$$-1 < \sigma < 0,\ \mathrm{Re}\,z \geqslant \sigma$$

Substituting (2) into (1), we obtain after changing the order of integration and evaluating the inner integrand by using the tables /8/

$$w_0 = \frac{1}{12i}\int_{\sigma+i\infty}^{\sigma-i\infty} \frac{(^1/_2\rho)^z\,dz}{\Gamma^2(1+{}^1/_2z)\sin{}^1/_2\pi z\sin{}^1/_3\pi(z+1)} \tag{3}$$

We find the value of the integral (3) by theory of residues. The singular points of the integrand are poles of first and second order. The simple poles are located on the real axis at $z = 6m$, $z = 6m + 4$ and $z = 6m + 5$ $(m = 0, 1, 2 \ldots)$, and the multiple poles at $z = 6m + 2$. Evaluating the residues at the points mentioned we arrive at the power series

$$w_0 = \frac{1}{6}\sum_{m=0}^{\infty}(-1)^m\left\{\frac{4}{\sqrt{3}}\left[\frac{\eta^{6m}}{((3m)!)^2} - \frac{\eta^{6m+4}}{((3m+2)!)^2}\right] + \frac{3\eta^{6m+5}}{\Gamma^2(3m+{}^7/_2)} + \frac{6}{\pi}\frac{\eta^{6m+2}}{((3m+1)!)^2}[\ln\eta - \psi(3m+2)]\right\} \tag{4}$$

where $\eta = \rho/2$, and $\psi(z)$ is the psi function which it is convenient to calculate by means of the recurrence relation

$$\psi(z+1) = \psi(z) + z^{-1},\ \psi(1) \approx 0{,}577216$$

The integral p_0 is evaluated analogously and is represented by the series

$$p_0 = \frac{1}{12}\sum_{m=0}^{\infty}(-1)^m\left\{\frac{4}{\sqrt{3}}\left[\frac{\eta^{6m}}{((3m)!)^2} + \frac{\eta^{6m+2}}{((3m+1)!)^2}\right] - \frac{3\eta^{6m+1}}{\Gamma^2(3m+{}^3/_2)} + \frac{6}{\pi}\frac{\eta^{6m+4}}{((3m+2)!)^2}[\ln\eta - \psi(3m+3)]\right\} \tag{5}$$

Since $\lim\psi(z) = \infty$ as $z \to \infty$, terms with the psi-functions have worse convergence in the expansions (4) and (5). Consequently, we investigate the convergence of the series from these terms. Taking into account the recurrence relationships for the psi-functions and factorials, we obtain

$$\lim_{m\to\infty}\{\psi(3m+5+j)((3m+1+j)!)^2[\psi(3m+2+j)((3m+4+j)!)^2]^{-1}\} = 0,\ j = 1, 2$$

i.e., according to the D'Alambert criterion the series (4) and (5) converge on the whole number axis. The rate of convergence is sufficiently high and keeping several of the first terms enables good accuracy to be achieved for small η. Thus for $m \leqslant 3$ the results of calculations by means of (4) and (5) and the data obtained by a numerical integration of the quadratures /2/ agree with 0,001 accuracy for $\rho \leqslant 4$.

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ON CERTAIN METHODS OF SOLVING SYSTEMS OF INTEGRODIFFERENTIAL EQUATIONS ENCOUNTERED IN VISCOELASTICITY PROBLEMS*

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The method of freezing proposed and given a foundation by A.N. Filatov, for systems of integrodifferential equations (IDE) of standard form /1-4/ is applied to IDE systems encountered in dynamic viscoelasticity problems. A numerical method is proposed for IDE systems, which is based on using quadrature formulas. A specific example is examined to compare this method with other known methods (the method of averaging and the method of freezing). Furthermore, a problem on the longitudinal vibrations of a viscoelastic rod in a physically non-linear formulation is investigated by the method of freezing in combination with a numerical Runge-Kutta method.

1. Let us consider an IDE system of the form

$$T_i^{\cdot\cdot} + \omega_i^2 T_i = f_i(t) + \mu X_i\left(t, T_1, \ldots, T_n, \int_0^t \varphi_i(t, \tau, T_1(\tau), \ldots, T_n(\tau))\, d\tau\right) \tag{1.1}$$
$$T_i(0) = T_{0i}, \quad T_i^{\cdot}(0) = T_{0i}^{\cdot}$$

Here $T_i(t)$ is the desired function of the argument t, $\mu > 0$ is a small parameter, f_i, X_i and φ_i are given continuous functions in the range of variation of the arguments, and the subscript i takes on the values $1, 2, \ldots, n$ everywhere.

By making the substitution

$$T_i(t) = C_{1i}\cos\omega_i t + C_{2i}\sin\omega_i t + \frac{1}{\omega_i}\int_0^t f_i(\tau)\sin\omega_i(t-\tau)\, d\tau \tag{1.2}$$

we can reduce system (1.1) to standard form. Applying the freezing procedure /1-4/ to the system obtained and taking account of relationship (1.2), we obtain after differentiation

$$T_i^{\cdot\cdot} + \omega_i^2 T_i = f_i(t) + \mu X_i\Big\{t, T_1, \ldots, T_n, \int_0^t \varphi_i\Big(t, t-\tau, T_1(t)\cos\omega_1\tau - \tag{1.3}$$
$$\frac{1}{\omega_1}T_1^{\cdot}(t)\sin\omega_1\tau - \frac{1}{\omega_1}\int_{t-\tau}^t f_1(s)\sin\omega_1(t-\tau-s)\, ds, \ldots, T_n(t)\cos\omega_n\tau -$$
$$\frac{1}{\omega_n}T_n^{\cdot}(t)\sin\omega_n\tau - \frac{1}{\omega_n}\int_{t-\tau}^t f_n(s)\sin\omega_n(t-\tau-s)\, ds\Big)d\tau\Big\}$$
$$T_i(0) = T_{0i}, \quad T_i^{\cdot}(0) = T_{0i}^{\cdot}$$

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